

Implication And McTaggart's Theory Of Relations

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This essay begins by covering some old ground on the peculiarities of material implication. Of course I hope to leave the reader with what may be some memorable impressions of those shortcomings. But the purpose for this is mainly to lay bare a consequence of material implication on the old controversy over external and internal relations, its bearing on McTaggart's concepts of implication and relations and to correct one of Broad's misleading accounts of the latter.

There is little doubt that McTaggart was aware of the work on the logic of relations that was greatly advanced in the latter half of the nineteenth century and continued into our own time.¹ His resistance to Bradley's monism, and his support for the existence of relations is clear in his essay "The Further Determination of the Absolute" of 1893, the same year that Bradley published *Appearance and Reality*. These two observations I take as basic presuppositions of my thesis. I further take them to show that McTaggart was unwilling to backtrack to the subject/predicate logic to which idealists are commonly assumed, rightly or wrongly, to be dogmatically devoted.

However, this should not be taken to mean that he adopts wholesale all that is advanced in these new logical doctrines. The most important exception which McTaggart takes is in regard to the nature of implication which he refuses to reduce to the truth-functional relation of material implication Russell used in *Principia Mathematica*. This is because implication is (1) essential to his theories of determining correspondence and intrinsic determination and (2) bears on the question of whether relations are internal or external.

Material implication is truth-functionally defined such that for any two statements so related, it is false that the first statement is

true and the second statement is false. Another way this is expressed is that either the first statement is false or the second is true. These may be more clearly seen when put symbolically as $\{-(p \cdot \neg q)\}$ and $\{\neg p \vee q\}$ respectively. That this is intended as a definition of implication is clear from Russell's opening remarks on the nature of material implication in *Principia Mathematica*.

It is in fact a theory of how one proposition may be inferred from another. Now in order that one proposition may be inferred from another, it is necessary that the two should have that relation which makes the one a consequence of the other. When a proposition q is a consequence of a proposition p , we say that p *implies* q .²

This is followed by an analysis of and justification for adopting material implication in place of any other sense of implication which, as Russell repeats from an earlier essay, is "very much more convenient than any of its rivals."³ Russell's case rests at bottom on the fact that it preserves one main condition of implication that allows the possibility of proofs; it does not allow a false proposition to follow validly from a true proposition. Again, in Russell's words, "what is implied by a true proposition is true."⁴ However, were this the only characteristic of material implication there would hardly be an issue. After all, this feature is common to every notion of implication. No sense of implication allows that a false consequent follows from a true antecedent. It would be revealing, however, for us to examine how well Russell's notion of material implication squares with his opening statements on the relation of implication cited above.

As is well-known, material implication brought with it its share of problems and controversy. More than can be summarized here has been written on it; the most important and familiar problems are those referred to as the paradoxes of material implication. These derive from its truth-functional definition which permits a false proposition

to "imply" any proposition; (true or false).⁵ According to this definition of implication, "Tolkien is an oriental author implies that he is the author of 'The Lord of the Rings,'" and "Tolkien is a British author implies that he is the author of *The Lord of the Rings*" are both legitimate implicational statements. This is so, even though the first statement has a false antecedent clause. Both compound statements are considered true on the basis that statements of material implication are false only when the antecedent is true but the consequent is false.

The peculiarity, here, consists in the fact that whether Tolkien is oriental or British is irrelevant to and does not imply in any ordinary sense that he wrote *The Lord of the Rings*. Even from the proposition that Tolkien is British it can not be inferred that he wrote *The Lord of the Rings*, even though both propositions are *in fact* true. Such a linkage between irrelevant terms is hardly what is ordinarily meant by implication. Yet, such a linkage is precisely what is permitted to pass as a relation of implication. This is due to the fact that all that matters, in Russell's view, is that the consequent be true. So long as the compound proposition does not link a true antecedent with a false consequent it is to be counted as implicational. If there is any relation here, it is not between the propositions or their meanings, but only between their truth values.

Other unusual properties and problems could be mentioned here. Two especially lucid discussions of these problems occur in Brand Blanshard's *Reason and Analysis*,⁶ and in the *Symbolic Logic* of Lewis and Langford.⁷ But these need not detain us. My purpose, after all, is not to banish material implication, but only to show that it is not an adequate substitute for what is usually meant by "implication" and thereby leave room for McTaggart's concept. It is enough, perhaps, to see the discrepancy between Russell's opening statement in *Principia* as to what is expected of the relation of implication and the results of taking material implication as an adequate substitute, a discrepancy that Moore described as "an enormous 'howler'."⁸ This

alone, however, does not supply reasons for rejecting material implication; for material implication preserves one key condition of all implicational relations.⁹ But, by the same token, material implication offers no grounds for abandoning the more common notion of implication adopted by McTaggart.

The relation of implication, according to McTaggart, is "an *undefinable* relation between propositions."¹⁰ It is not concerned merely with a set of possible combinations of truth values, and therefore is not further reducible to either a form of conjunction or disjunction as in the case of material implication. He describes implication in terms of certain conditions which preserve *modus ponens* and *modus tollens*, two fundamental forms of inference. This is brought out in the passage below:

P implies Q when (1) if I know that the relation holds between P and Q, and know P to be true, I am justified by this knowledge alone in asserting Q to be true, and when (2) if I know that the relation holds between P and Q, and know Q to be false, I am justified by this knowledge alone in asserting that P is false. From this, of course, follows the proposition that Q must be true or P false.¹¹

More may be gathered from a key remark McTaggart makes about his method. But in order to appreciate the salience of that remark, it will be worthwhile to examine the statement above. At first, we might fault McTaggart for not explaining further. Indeed, Broad complains that McTaggart's statements given above are "neither clear nor satisfactory."¹² We might readily agree, if McTaggart is taken to be *defining* implication. Also, at first glance, McTaggart's account doesn't seem sufficient to distinguish his view from Russell's on the nature of implication. But these faults can be remedied.

Let us begin by stressing the key point of Broad's complaint. He admits that McTaggart never accepted Russell's reduction of impli-

cation to material implication. (This much can be taken for granted given the aims of McTaggart's metaphysics.) However, Broad holds that McTaggart does not make this clear. According to Broad, *entailment* is what McTaggart has in mind in his discussion of implication.¹³ This is a welcome change of terminology; for "entailment" carries the sense that P entails Q if and only if P is inconsistent with not-Q. Thus, for example, "the ball is red but it is not colored" is inconsistent.

Now, Broad's clarification is helpful. But he is mistaken to accuse McTaggart of having neglected this sense of entailment. For it is precisely this sense to which McTaggart appeals in describing his method. His way of stating the relation is very explicit; i.e., that "there is a contradiction between asserting the first [characteristic] to be true and denying the second [characteristic] to be true."¹⁴ Thus, taking this passage together with the passage on implication we get the sense of entailment that Broad insists on.

Along these lines, the difference between McTaggart's and Russell's conceptions of implication becomes obvious. That there is an inconsistency between the affirmation of one statement and the denial of another is to state a much stronger relation between the two than merely that it is not the case that P and not-Q.¹⁵ To suggest otherwise would be to say: (1) As a *matter of fact* it so happens that it is not true that the ball is red but it is not colored.¹⁶ This interpretation fails to convey the full force of the statement which is: (2) It *could not* be true that the ball is red but not colored. Of course, line (1) follows from line (2), but not the reverse. Thus, if P is inconsistent with not-Q, it follows that not P and not-Q, i.e., $\{-(p \rightarrow q)\}$. If all that is known is not P and not-Q, it doesn't follow that P is inconsistent with not-Q. It is in this sense that the relation described by McTaggart is stronger than material implication.

So far, our discussion of implication has concentrated on the purely logical issue of the difference between Russell's and McTaggart's respective notions of implication. If I may sum up the

results of the foregoing in a single phrase, it is that the two senses of implication are compatible but not substitutable. Russell's is obviously the wider but weaker sense attempting to define implication in terms of only one of its conditions. McTaggart's is the narrower and stronger sense of entailment, requiring not merely the appropriate combinations of truth-values, but also the condition of consistency between the meaning of related propositions, a condition based on the meaning of the related terms.¹⁷ This difference between the two conceptions of implication has important consequences, being at the root of the distinction between internal and external relations, and is critical to understanding McTaggart's theory of relations. To show these connections, a brief but explicit statement of these consequences should be given.

The importance of the difference between material implication and entailment consists in the following: An analysis of two (or more) propositions according to the condition of material implication represents the relation between the propositions in a way that favors the view that the terms described by the propositions are externally related; i.e., the presence or absence of the relation is of no consequence to the terms related. In the case of entailment, the relation between the terms described in the propositions is internal, or more properly intrinsic. As we saw above, according to Russell's view, the meaning and even the *actual* truth or falsehood of two (or more) propositions is irrelevant to their being related by material implication, as long as it is not the case that the antecedent is true and the consequent false. Thus, the propositions are independent of each other as well as of the relation. These are precisely the features of material implication that are reflected in the theory of external relations.¹⁸ Thus, having taken material implication as an adequate substitute for the more common conception, it is not surprising to find Russell favoring the doctrine of external relations.

On the entailment view accepted by McTaggart, the meanings of the related terms of the propositions and the relation of consistency

between them are not independent of each other. In regard to the theory of internal relations, it is this feature of mutual dependency between the propositions and the relation between them from which the theory derives its support. Thus, having accepted this view of implication, McTaggart's arguing in favor of internal relations is to be expected. But how does McTaggart make use of entailment in his theory of relations? To this question, we now turn.

The most conspicuous use of relations in McTaggart's metaphysics is in his theory of Determining Correspondence, which relations consist of a one/one correspondence and the relation of intrinsic determination. The function of correspondence is twofold: (1) It is a relation such that the parts of substances sequent in a series correspond one/one with the parts of precedent substances, and (2) in some cases there is a cross-correspondence (again, one/one) of a part of one substance to that of another. Considered apart from the other conditions of Determining Correspondence, the one/one correspondence does not in itself function as a relation of intrinsic determination whereby a unique description of one substance determines (at least part of) a unique description of another. Rather, the correspondence amounts to a mere pairing of an individual substance *qua individual* with another individual substance *qua individual*. In other words, it no doubt is the case that the two substances in question are more fully and rigorously related, but they are at least paired one/one.

On the other hand, intrinsic determination is not a mere pairing of substances insofar as they are individual; it is a relation between qualities. In the case of two substances, intrinsic determination is a relation between the unique descriptions of each; i.e., it is a relation such that the characteristics of one substance are a condition for (some of) the characteristics of another substance. For example, take two substances, X and Y, where X and Y are father and son: X's having the quality of being a father is the condition of Y's having the quality of being a son. Since intrinsic determination is modeled on entailment, the descriptions of two substances are related by intrinsic de-

termination when one substance having a certain quality is incompatible with the other substance lacking a certain quality. Again, Y's being a son is *inconsistent* with it being *not* the case that some other substance, X, is a father. Of course, it is also true in the case of one substance alone that two of its qualities can be related by intrinsic determination, e.g., X's being a father is inconsistent with X's being not male. Much of this should be familiar. I wish to emphasize that, as characterized above, intrinsic determination conforms to the entailment notion of an internal relation; i.e., if two terms, P and Q, are related such that if P is inconsistent with not-Q, then P and Q are internally related.

It is worthwhile to note that this is one conception of internal relations accepted by Moore in his famous paper,¹⁹ and even accepted by other philosophers who are commonly regarded as being of an anti-idealistic persuasion.²⁰ To be sure, neither Moore nor any of the others are willing to generalize this conception to *all* relations; they insist, and rightly so, that it is true only of *some* relations. However, in taking this line, they are in agreement with McTaggart who holds that only some are relations of intrinsic determination.

Above we may have shown the connection between entailment, internal relations, and McTaggart's sense of intrinsic determination. But what is its ontological import in his system? Within the limits of this essay, I can give only an incomplete answer. Relations of intrinsic determination with the one/one correspondence constitute relations of determining correspondence between the members of groups of substances; more precisely, a community of perceivers mutually perceiving each other and themselves. Since perceivers and their perceptions are the only substances McTaggart recognizes, the relations enabling groups of substances to constitute a community are perceptual relations of intersubjectivity and self-consciousness. If McTaggart's theory of relations is correct, it provides one of the most rigorous theories of intersubjectivity, demonstrating the necessary relation between self-awareness and awareness of the other. But this is a topic for another time.

Notes

1. William Kneale and Mary Kneale, *The Development of Logic*, (Oxford: Clarendon Press, 1968), p. 427.

2. Alfred North Whitehead and Bertrand Russell, *Principia Mathematica To * 56*, (Cambridge: University Press, 1962), p. 90.

3. "The Theory of Implication," *American Journal of Mathematics*, Vol. 28, 1906, p. 161. Also see *Principia Mathematica To * 56*, p. 94.

4. *Principia Mathematica To * 56*, p. 94.

5. The truth table for material implication may be found in any common logic text, but is given here for convenience (" $>$ " = materially implies):

p	q	p	$>$	q
T	T	T	T	T
F	T	T	T	T
T	F	F	F	F
F	F	F	T	T

It was this feature that moved C.I. Lewis to develop his system of "strict implication." cf., his *Survey of Symbolic Logic*, Ch. V. However, Lewis' system bears a similar paradox; i.e., an impossible proposition implies every proposition and a necessary proposition is implied by every proposition. Cf., Kneale and Kneale, *The Development of Logic*, p. 549. For this reason it is not advisable to uncritically assume that McTaggart's concept of implication is captured in Lewis' strict implication. Also see: William Kneale, "Truths of Logic," *Proceedings of the Aristotelian Society*, N.S., Vol XVI, 1945-46, p. 216; Everett J. Nelson, "Intensional Relations," *Mind*, Vol. XXXIX, Oct.

1930, pp. 445-446. Most recently, Edward Pols has done some very important work on the nature of logical implication. I unfortunately learned of his work too late to avail myself of his results to enrich this essay (see n. 9 below).

6. LaSalle, Ill.: Open Court Publishing Co., 1964, Ch. IV, pp. 158-169.

7. New York: Dover Publications, Inc., 1952, Ch. VIII, 235-242. Other recommended works on this topic include: (1) Stephan Korner, *Conceptual Thinking*, (New York: Dover Publications Inc., 1959), Chs. V, VII, and X; (2) A.C. Ewing, *Idealism: A Critical Survey*, (London: Methuen & Co., Lt., 1961), pp. 17n.3, and 252-258.

8. G.E. Moore, "External and Internal Relations," in *Philosophical Studies*, (Paterson, N.J.: Littlefield, Adams & Co., 1959), pp. 303-4, cf. pp. 296-7 for his explanation.

9. In a recent essay, Edward Pols says that material implication "expresses a certain minimal sense that all such connections have in common." See his "Logical Implication and the Ambiguity of Extensional Logic," *The Review of Metaphysics*, Vol. XLVII, No. 2, Dec. 1993, p. 237. Unfortunately, Pols' article had not been published when I submitted this essay for the 1994 NMWTPS Conference. To include more of his insights and results in the text of my essay would lengthen it beyond the journal's restrictions. But his work is important and confirms some of my claims. Therefore, I merely refer the reader to key passages in Pols' essay.

10. *The Nature of Existence*, (Grosse Point, Michigan: Scholarly Press, 1968), Vol. I, p. 110, (hereafter *NE*). My emphasis.

11. *Ibid.*

12. *Examination of McTaggart's Philosophy*, (New York: Octagon Books, Farrar, Straus & Giroux, Inc., 1976), Vol. I, p. 193.

13. *Ibid.*

14. *NE*, Vol. I, p. 46.

15. G.E. Moore also distinguishes between entailment and material implication in his essay "External and Internal Relations," *loc. cit.* Broad, no doubt, was aware of this.

16. The emphasis is upon the contingency inherent in this interpretation. The point has been put convincingly by Everett J. Nelson in "Intensional Relations," *Mind*, Vol. XXXIX, Oct. 1930, p. 441 ff.

17. Cf. Edward Pols *op. cit.*, pp. 238, 241, 244. Throughout his article, Pols appeals to *meaning* and *relevance* to argue his case for entailment and intensional logic. But, he also develops what he calls a protocalculus that symbolically represents the relation of entailment in a quite rigorous manner.

18. Cf. Bertrand Russell, "The Monistic Theory of Truth," *Philosophical Essays*, (N.Y.: Simon & Schuster, 1966) pp. 139-140; and "The Philosophy of Logical Atomism," *Logic and Knowledge*, R.C. Marsh (ed.) (N.Y.: The Macmillan Co., 1971), pp. 201-202, cf. pp. 209-211.

19. G.E. Moore, "External and Internal Relations," pp. 277, 286 ff. Cf. Pols, *op. cit.*, pp. 238, 246-249, where he argues, as I have, that the consistency/inconsistency condition is at the root of entailment; though Pols makes no excursion into the metaphysics of relations.

20. e.g., Gilbert Ryle, "Internal Relations," *Proceedings of the Aristotelian Society*, Supp. Vol. XIV, July, 1935, p. 155.